

$$\left\{ \begin{aligned}
 T_{\text{trans}}(E < V_0) &= \frac{1}{1 + \left(\frac{k_1^2 + k_2^2}{2k_1 k_2}\right)^2 \text{sinh}^2(k_2 L)} \\
 T_{\text{trans}}(E \geq V_0) &= \frac{1}{1 + \left(\frac{k_1^2 - k_2^2}{2k_1 k_2}\right)^2 \sin^2(k_2 L)}
 \end{aligned} \right.$$

T versus energy: $E = \frac{\hbar^2 k_1^2}{2m} \rightarrow k_1 = \frac{\sqrt{2mE}}{\hbar}$

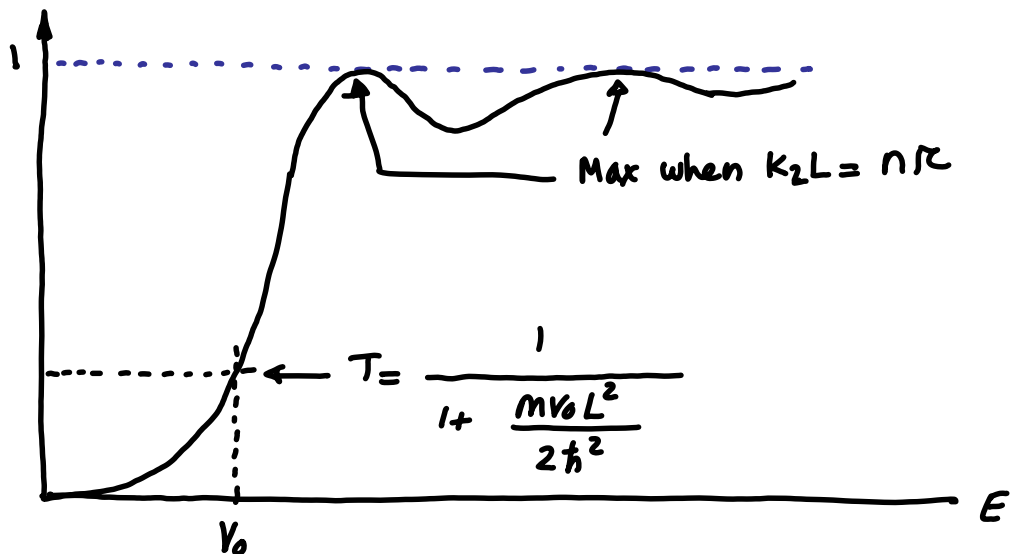
when $E < V_0$: $-E + V_0 = \frac{\hbar^2 k_2^2}{2m} \rightarrow k_2 = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

when $E > V_0$: $E - V_0 = \frac{\hbar^2 k_2^2}{2m} \rightarrow k_2 = \frac{\sqrt{2m(E - V_0)}}{\hbar}$

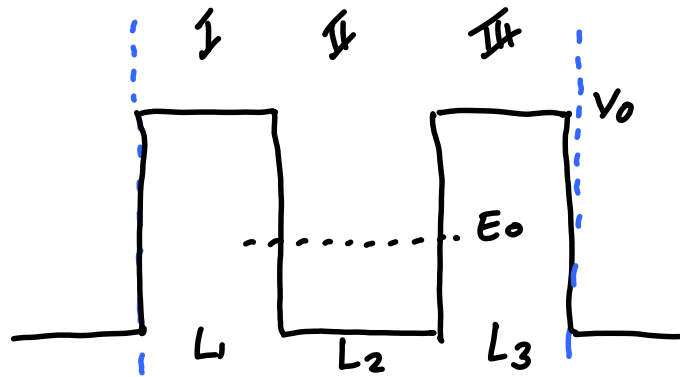
Substitute k_1 & k_2 in $T \Rightarrow$

$$T(E < V_0) = \frac{1}{1 + \frac{V_0^2}{4E(E - V_0)} \text{sinh}^2(k_2 L)}$$

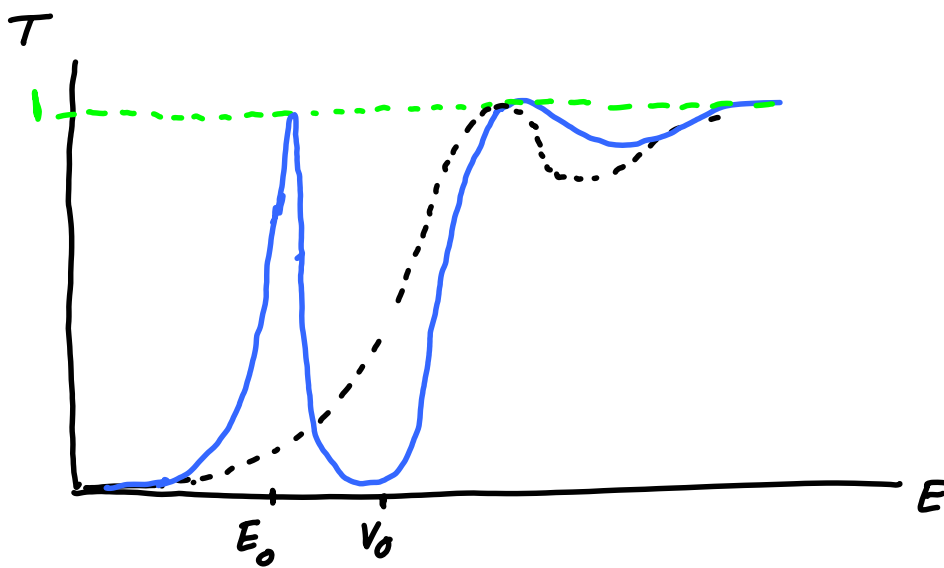
$$T(E > V_0) = \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} \sin^2(k_2 L)}$$



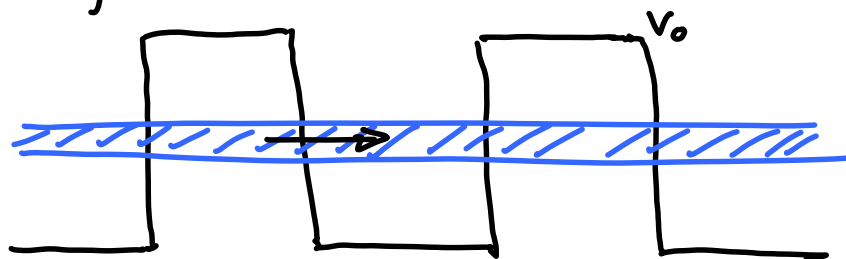
Resonant tunneling



$$P = P_S(V_0) P_f(L_1) P_S(-V_0) P_f(L_2) P_S(V_0) P_f(L_3) P_S(-V_0)$$



Miniband formation in SL's:



Bloch's Theorem

for a free electron in 1D space we have:

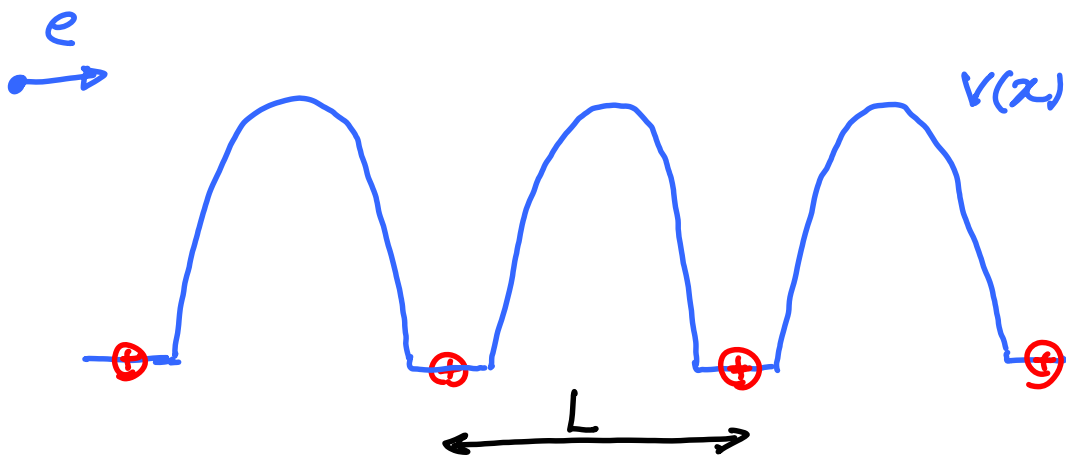


$$-\frac{\hbar^2}{2m} \psi'' = E\psi \Rightarrow \psi(x,t) = A e^{ikx} e^{-i\omega t}$$

$$\Rightarrow |\psi(x,t)|^2 = A^2 \quad \text{So the probability}$$

of finding the electron is uniform everywhere.

Now we add a periodic potential $V(x)$:



$$V(x) = V(x + nL) \quad \text{which means it's periodic}$$

This is the case for an electron moving in a crystal.

To find ψ , solve the Schrödinger equation:

$$\hat{H}\psi = E\psi$$

$$-\frac{\hbar^2}{2m} \psi'' + V(x)\psi = E\psi$$

Since $V(x)$ is periodic, $|\psi|^2$ is periodic:

$$|\psi(x)|^2 = |\psi(x+nL)|^2$$

The **Bloch's theorem** states that ψ has the following form:

$$\psi(x,t) = U(x) e^{ikx} e^{-i\omega t}$$

and $U(x)$ is periodic: $U(x) = U(x+nL)$